The Impact of Macroprudential Policies on Systemic Financial Stability

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Abstract

Financial networks are vulnerable to contagion and systemic risk. A financial network's stability, in particular, depends on the resilience of Systemically Important Financial Institutions (SIFIs). In this article, we introduce a metric to identify SIFIs. The metric is based on an explicit characterization of the system's equilibrium behavior. We consider different mechanisms of contagion: common shocks, domino-fashion sequential defaults, and combinations thereof, emerging from random institutes or SIFIs. We measure the stability of the financial system by means of a system-wide leverage ratio. Furthermore, we analyze to what extent macroprudential policies can reduce the transmission of shocks in the system. Our findings show that none of the existing regulatory policies—when considered individually—can prevent or mitigate a systemic collapse.

Keywords: Financial Networks, Systemic Risk, Financial Stability, Macroprudential Policy

1. Introduction

The recent financial crisis has shed light on the importance of the degree of complexity of the financial system for systemic stability. The elements of such a system are strongly connected such that the entire system is non-decomposable. The structure of this system dynamically changes, and the system possibly moves away from a stable state. To design proper regulatory and supervisory mechanisms to stabilize the financial system, we need to therefore understand its dynamics and how the system changes over time.

We study the stability of an interbank system as a complex network. A network approach may yield a better understanding of the complexity of financial systems. Network theory provides tools to assess the stability of a system and to identify systemically important players. Network models can explain the dynamics of exposure transmission through the system and quantify systemic stresses.

In our network setting, nodes of a network might represent financial instruments or financial institutions such as different types of banks, insurance companies, and hedge funds. The connections or links might represent transmissions of financial stress at the time of default or the dependency of institutions holding common assets. The most comprehensive and complete theoretical model of a financial network was studied by Allen and Gale (2000), who extended the banking model of Diamond and Dybvig (1983) into a full network. Allen and Gale (2000) examined the role of networks within a standard three-date banking model with liquidity distress and liquidation costs at intermediate dates and showed how small shocks, which primarily affect only a small number of institutions, spread through the whole system. A real interbank network was studied by e Santos and Cont (2010), who analyzed the role of network approaches in the estimation of systemic risk by defining a metric to identify causes of contagion using a data set of bilateral exposures of Brazilian financial institutions. The interconnectedness of financial systems has been emphasized in some recent studies: Glasserman and Young (2015) and Amini et al. (2012a) applied a network-type model to analyze the likelihood of transmission of financial exposure in interbank networks. Poledna et al. (2015) considered a financial system as a multi-layer network to investigate systemic risk. Lee (2013) set up a simple banking system model and found that inequality in liquidity positions across banks is more likely to intensify liquidity shortages in insolvent banks.

In our study, we measure systemic risk using network science. In the aftermath of the 2007–2008 financial crisis, there have been numerous studies on systemic risk. The concept of systemic risk might be defined as the propagation of shocks in the financial system as a result of exogenous or endogenous events that have a significant impact on the economy. Systemic risk has a huge impact on financial system resilience and can inflict severe damage to the economy. There have been various attempts to measure systemic risk. The Handbook of Systemic Risk Fouque and Langsam (2013) collected various aspects of systemic risk that impact financial markets. There exist many works on the measurement of systemic risk by means of network science. In a network approach, the collapse of an individual firm or groups firms can cause a cascading failure and bring down the entire system. From this perspective, systemic risk refers to the risk emerging from the interdependencies within a system. Eisenberg and Noe (2001) remain one of the pioneers of measuring systemic risk with a network approach. They applied fixed-point theory and found a market-clearing equilibrium based on a clearing payment vector with proportional sharing of risk among the counterparties in the network.

Adrian and Brunnermeier (2011) proposed a metric (COVaR) to measure co-dependence in the tails of equity returns of financial firms. The primary idea is to find institutions with high tail risk, and (thus) greater exposure to risks from systemic liquidity hoarding. This metric is based on a quantile regression method using equity price and balance sheet information. Another tail measure technique, suggested by Acharya et al. (2010), is called Systemic Expected Shortfall (SES), which measures using an empirical sampling-scaling method how much an institution, at the aggregate level, might be undercapitalized during a systemic financial crisis. This metric too is based on the information on equity prices and balance sheets. Due to the limited historical data and a very small number of extreme financial events, a tail measure approach might not measure the magnitude of a financial crisis correctly.

To identify Systemically Important Financial Institutions (SIFIs) in a banking system, we apply a network-based model in a single-period framework. Note that SIFIs can be banks or other financial institutions, and may also be non-financial institutions whose failure can cause a systemic collapse. Battiston (2012) developed a metric called DebtRank to measure the systemic impact that is inspired by the feedback-centrality of a network. The SinkRank metric was introduced by Soramäki and Cook (2013) and predicts the severity of disturbance caused by the failure of an institution in a payment system. Huang et al. (2009) proposed a framework for measuring systemic risk in a set of major financial institutions.

We focus on two main contagion mechanisms. The first mechanism has been studied by Gai and Kapadia (2010), Nier et al. (2007), and Upper

(2011) who modeled shocks that could spread throughout the entire system or a part of it in a domino fashion due to interlinked financial exposures. Brunnermeier (2008) and Gai et al. (2011) considered a second channel of contagion, namely common shocks (as caused by exogenous factors) affecting the entire system. Another transmission channel is asset price contagions, that is, co-movements in asset prices caused by a common shock in an illiquid market. Adrian and Shin (2010), Cifuentes et al. (2005), and Coval and Stafford (2007) developed theoretical models of the impact of asset price contagion on financial systems.

We consider the stability of an interbank system and how to measure it in a network-framework. We compute the stability of a banking system-based financial leverage ratio defined on the basis of the degree of interbank network connectivity and exposures. Financial stability refers to the ability of the system to constantly supply payment services and credit, which are necessary for economic growth (Beck (2014)). It is essential to clarify the concept of financial stability and its underlying factors. Knowing these factors is critical to designing appropriate regulatory and supervisory mechanisms. There is no standard framework to assess the stability of financial systems and to examine financial regulation policies. The health status of financial institutions does not indicate the stability of a system and similarly, individual defaults do not necessarily lead to system instability. Financial market fluctuations, group failures of financial and non-financial institutions, lack of intermediation services, and other issues have systemic consequences on the economy. Nevertheless, it is not well understood which of these sources contributes more to system instability. To measure financial stability, in a broader perspective, we need to identify factors that have a substantial impact on the financial system. Some studies attempt to measure systemic stability as an aggregation of stability at the institutional level. Financial stability in an interconnected complex system has been studied by Battiston et al. (2012), Amini et al. (2013), and (in a recent study by) Acemoglu et al. (2015). These works used a complete network of exposures wherein all nodes have direct financial interactions and obtained a network equilibrium. In their network setting, all nodes are homogeneously connected. Elliott et al. (2014) considered the heterogeneous setup of an interbank exposure network and studied the effect of highly interconnected nodes on contagion stress through the network. This study found a non-linear relation between the diversification of the exposure and the number of default nodes. However, these metrics are defined in a closed banking system without considering the important role of system interconnections.

The main aim of this article is to arrive at a better understanding of the factors associated with systemic collapse. We hope to clarify the interrelationship between the individual characteristics of financial institutes and the financial system, and make recommendations designed to enhance system stability. We analyze the impact of macroprudential policies on systemic financial stability, and how these policies diminish the effects of shocks in different network configurations. Systemic crises are rare but might severely damage entire economies. To prevent systemic crises while facilitating economic growth, it is essential to improve the resilience of the financial system, without excessively costly regulation and higher microprudential capital standards.

In other words, it is vital to choose the appropriate aggregate or particular capital- or liquidity-based prudential regimes. In the models by (Gertler et al. (2012)) and (Wagner (2010)), microprudential policies focus on the risks and resilience of individual institutions with the objective of minimizing failure at the individual level. Hanson et al. (2010) comprehensively reviewed macroprudential policies and how these policies might be designed.

In this paper, we study the stability of an interbank network as a complex system. We review the sources of a systemic collapse. Using fixed-point theory, we identify Systemically Important Financial Institutions (SIFIs) in the financial network by considering the explicit characterization of the equilibrium behavior of a banking system. We discuss the stability of a financial system and how to measure it in an interbank network. We simulate a random financial network based on the parameters of a real financial network. In our simulation, we focus on different mechanisms of contagion: common shocks and domino-fashion sequential defaults and combinations of both mechanisms started by random institutes or SIFIs. We apply various micro- and macroprudential policies to reduce systemic liquidity surcharges and mitigate the effect of shocks within different network configurations. After applying these policies, we assess the stability of the system and identify the weakest link or node in the network.

Our results highlight that when designing suitable regulatory and supervisory regimes to identify financial stability, we need to consider the mechanism underlying financial stability to ensure continual provision of financial services for economic growth. Policymakers should use specific indicators to identify SIFIs and accordingly propose macro-prudential tools for an unseen systemic crisis. A weakness of Macro-prudential tools is to set regimes tightly in regular times for both SIFIs and non-SIFIs; this, however, might have an effect on the efficiency of banks during normal times. Our numerical results indicate that it is vital to choose suitable aggregate or specific macroprudential regimes for SIFIs and non-SIFIs in a system.

The remainder of this paper is organized as follows. Section 2 schematically characterizes a bank's balance sheet, describes the network approach to financial systems, and explains the concept of systemic risk and the mechanism of financial contagion. We introduce a metric to identify SIFIs in a financial system in section 3. The mechanism of financial contagion is explained in section 4. The concept of financial stability is the subject of section 5. We provide common issues associated with regulation of financial institutions in section 6. Section 7 presents a numerical simulation of the model and demonstrates some experiments to investigate how regulatory regimes affect system stability. Section 8 concludes the paper with further remarks. Further discussions on network science and the proofs of the propositions are given in the appendix.

2. Balance Sheets, financial networks and systemic risk

2.1. Balance Sheet Analysis

In simple terms, a bank's balance sheet contains information on how much the bank owns, how much its counterparties owe to the bank, how much the bank owes to its counterparties, and on the size of its equity. The structure and size of the liabilities and assets on a balance sheet can be a significant source of the weakness of financial institutions in times of crises. In this study, we concentrate on the key characteristics of banks' balance sheets. Typically, the values in banks' balance sheets refer to the values of assets and liabilities that are decomposed into interbank and non-interbank quantities depending on credit and/or counterparty debt. A complete balance sheet investigation requires detailed information on the size, maturity, and structure of the assets and liabilities.

Table 1 schematically characterizes a bank's balance sheet. On the left-hand side of a bank's balance sheet, assets are listed as fully liquid assets (A^L) such as central bank reserves or government bonds. Other types of assets include fixed assets (A^F) such as mortgages or individual corporate loans, and collateral assets (A^C) that can be used as collateral in repo transactions. On the right-hand side, liabilities are usually listed as interbank liabilities (i.e., unsecured financing borrowed by a bank from its counterparties). Secured repo liabilities, collateral borrowing, and external financial sources like retail deposits (L^D) are other classes of liabilities.

Assets	Liabilities
Interbank Assets A^{inte}	Interbank Liabilities L^{inte}
Fixed Assets A^F	Deposits L^D
Collateral Assets A^C	
Liquid Assets A^L	
...	Capital K

Table 1: A stylized balance sheet.

For some banks, balance sheet information on the size and maturity of liabilities and assets is incomplete or not publicly available. The uncertainty of asset quality and the need to turn over liabilities can lead to liquidation risk in the short term and solvency issues in the long term. The portion of the asset value that is used as collateral is called the haircut.

The value of one-unit of a collateral asset is given by

$$V^c = (1 - h - h_i)A^c \tag{1}$$

where h_i is the risk-based individual haircut and h is the aggregate haircut. A bank can borrow a maximum of V^c from the central bank against one unit of the collateral asset (A^c). Fecht et al. (2015) showed that a collateral's haircut depends on the collateral's type and coupon type. Haircut size is often related to the asset's liquidity risk. The higher the value of the haircut, the safer the position of the lender, and the better protected the lender is against losses arising from sales of collateral due to counterparty defaults.

2.2. Financial Networks

Studying the complexity of financial systems shows that the network of financial institutions' interactions plays an important role in determining the systemic importance of individual institutions in a system. Therefore, it is necessary to devise a theory of how such systems work and how they are built. The growing literature on financial network formation addresses a variety of questions: how the systems are constructed, how asset values are allocated to the players, and finally, how counterparties are chosen. The network approach plays a vital role in modeling the information and financial flows through a system and in explaining the dynamics of exposure transmission through the system. From this viewpoint, systemic stress can be quantified by analyzing the dynamic structure of a network.

Depending on how relationships are defined, networks can take different shapes. A non-directed graph is the simplest form of a network, where two actors are either joined or not. Another type of system is constructed using nodes and directed links between them. This type of network is central to our model, which aims to determine the most systemically critical nodes in a network.

In our set-up, a complex financial network consists of nodes representing financial institutions and links representing their financial dependencies. In a stylized network, $V_{(i\in\{1,\dots,N\})}$ is a set of financial players who are connected in a network relationship where links (E^{ij}) represent economic activities. Formally, our set-up is a directed network.

In this network setting, nodes represent financial institutions, such as different types of banks, insurance companies, and hedge funds, and the link E_t^{ij} represents financial stress, for example, at the time of institutional default. Banks use capital buffer K_t^i against financial stress. In this weighted-network setting, we define a matrix W as an adjacency matrix with entries that represent the strengths of nodes relative to each other if there are interbank relations between them; the entries are zero if no such relations exist. In our banking model, the strength of a node is described by relative financial exposure to its counterparties.

A common way to describe a network is by means of the distribution of the degree of nodes. The in-degree of a node represents the number of creditors: $degree_i^{in}? = \#\{j?W|w_{ji} > 0\}$. Similarly, the number of debtors can be explained by the number of outgoing links, $degree_i^{out}? = \#\{j?W|w_{ij} > 0\}$. The network's degree distribution, $P_{degree}(degree^{in}, degree^{out})$, is defined as the distribution of nodes in a given network, where the in-degree is denoted by $degree^{in}$ and the out-degree by $degree^{out}$.

2.3. Systemic Risk

Systemic risk refers to the propagation of shocks through the financial system as the result of exogenous or endogenous events that have a significant impact on the economy. An individual institution may be exposed to different types of financial risk: market risk, credit risk, liquidity risk, and operational risk. These classes of risk appear at individual levels and can be mitigated by applying microprudential policies. On the other hand, systemic risk might sometimes not be mitigated through diversification. The foremost source of systemic risk is default contagion in interbank markets as a result of economic distress, which spreads throughout the financial system in a domino fashion or due to the effects of common asset exposure. Some empirical studies show that the concentration of a large number of institutions on common assets and a default in one institution might lead to the insolvency of that institution's counterparties who hold the same assets. The flash crash in May 2010 showed that the mispricing of assets can be an additional source of a systemic liquidity crisis. Thus, the measurement of systemic risk is not just the quantification of the exposures of bank portfolios to risk factors, but also that of the links among banks/institutions to determine exposures to counterparties. One of the leading factors of systemic risk is the contagion of monetary distress in the financial system in a domino fashion. Measuring and quantifying the spillover effects that arise from direct and indirect connections between firms is complicated. A network approach is well suited for dealing with the dynamic propagation of shocks within the economic system and the heterogeneous characteristics of the financial system. The connectivity and centrality of a financial network can play different roles in absorbing or amplifying stresses during crisis and non-crisis periods. It is essential to study the mechanisms that can lead to systemic collapse and identify the metrics for systemic risk.

3. Systemically Important Financial Institutions (SIFIs) and their Identification

Banks, insurance firms, and other financial institutions, whose breakdown may cause an economic crisis, are systemically important if they are connected to other institutions via main transmission channels. Such institutions might have relations with other firms that are also indicated as systemically important institutions. It is essential to recognize SIFIs in a financial system, for these firms have become the main targets of legislation and regulation. New regulations require the financial institutions recognized as SIFIs to have contingency plans for possible systemic failures and to hold higher capital or liquidity buffers than non-SIFIs. These regulations try to prevent SIFIs from becoming "too big to fail" or "too connected to fail" and from running into financial trouble in crisis events.

The Financial Stability Board (FSB) and G20 periodically update the list of international financial institutions, which are strong transmission channels to many other institutions, called Global Systemically Important Financial Institutions. Several factors have been used to identify these institutions. The first one is total assets and revenues, representing the size of an organization. Additional factors are revenues derived from non-home countries and holdings of intra-financial assets and liabilities as a measurement of the interconnectedness of an institution.

Financial institutions are typically vulnerable to instability since mostly they finance long-term assets with short-term liabilities or interbank liabilities. When a firm experiences monetary difficulties, its counterparties can become unwilling to maintain financial relationships with it. As a result, the firm suffers from a liquidity problem or a solvency problem, and it imposes stress to its counterparties. As systemic risk can also arise from other (i.e., non-financial) sectors, even non-financial firms need to be taken into account as systemically important institutions.

We use a network-based model of the interbank system to identify the SIFIs in a banking system. Later, we examine the mechanism of liquidity shortages, which might spread through the financial system via interconnections; we use this model for evaluating the effectiveness of our macroprudential scenarios to assess financial stability.

By looking at a snapshot of the financial system, we define an index to measure the impact of the distress of institutions on the entire banking system in a recursive way. We use this index to identify the systemically important institutions associated with the structure of dependencies of financial institutions.

The proposed model is a single-period model of interbank activities. We use a stylized balance sheet (Table 1) to reflect an actual balance sheet of all types of banks involved in both commercial and investment banking. The stylized balance sheet of bank (i) consists of assets listed as fully liquid assets $(A^L)^i$, fixed assets $(A^F)^i$, and collateral assets $(A^C)^i$. The right-hand side of the balance sheet of bank i lists unsecured interbank assets from bank i to bank j, $(A^{int})^{ij}$, unsecured interbank liabilities of bank i to bank j, $(L^{int})^{ij}$, and repo deposits, $(L^D)^i$.

The capital of a bank is defined as the difference between its asset value and its liabilities. It can be considered as the margin when a bank liquidates its assets, and the amount that would be received by its creditors. Banks determine the optimal capital level by examining how much capital must be provided to prevent insolvency risk. Capital buffers are critical factors in forcing liquidation and preventing contagion. Interbank contagion can be restricted if the banks are well capitalized in response to possible exogenous shocks. In our model, capital buffer can be expressed as

$$K^i = (A^F)^i + (A^C)^i + (A^L)^i + \sum_j (A^{int})^{ij} - \sum_j (L^{int})^{ij} - (L^D)^i. \tag{2}$$

Each institution (node (i) in the network) uses capital K^i against financial stress. We consider a directed weighted network where the nodes represent financial institutions that have financial exposures to their counterparties. The exposure link (E^{ij}) defines the interbank exposure of a bank, and comprises its unsecured interbank assets and liabilities. It is supposed that interbank assets of an institution are distributed equally over its counterparties.

$$E^{ij} = (A^{int})^{ij} - (L^{int})^{ij}. \tag{3}$$

A bank becomes insolvent when its current liabilities exceed its current assets. Gai and Kapadia (2010) defined the solvency condition as follows:

$$(1-\phi)\sum_{j} E_t^{ij} + (A^F)^i + (1-h-h_i)(A^C)^i + (A^L)^i - (L^D)^i > 0, \tag{4}$$

where ϕ is the portion of banks with obligation to bank i when it defaulted. The risk-based individual haircut h_i and aggregate haircut h are portions of assets used as collateral to protect lenders against counterparty defaults. Gai and Kapadia (2010) assumed that the counterparties of a defaulted bank lose their interbank assets held by that bank. Therefore, we can express the solvency condition of bank (i) as follows.

Definition 1. ***Solvency Index***
The solvency index is defined as a measure of an individual institution's ability to meet its debt and other obligations:

$$Solvency_i = \frac{(A^F)_t^i + (1-h-h_i)(A^C)^i + (A^L)^i - (L^D)^i}{\sum_j E^{ij}} > (1-\phi). \tag{5}$$

This index is a form of leverage that can also be expressed as a leverage multiple. This means that whenever a node i defaults, its counterparty node j faces loss E^{ij}, and without the required capital buffer, it becomes insolvent. Given this, we define the vulnerability weight as the ability of a node to control the transmission of shocks of its counterparty or when it is under distress or has defaulted.

Definition 2. ***Vulnerability Weight***
In an interbank network, the vulnerability weight of a node is defined as the portion of capital that is reduced because of the solvency risk of its counterparty:

$$w_{ij} = \frac{E^{ij}}{K^i}. \tag{6}$$

3.0.1. Systemic Impact Index

Based on the underlying factors of the solvency condition of a bank, the health status of a bank depends on the status of its counterparties. If one or some of its counterparties (which are well connected) default, the bank is unable to accomplish its long-term obligations; consequently, it becomes insolvent. Therefore, the relative impact of one firm on its counterparties can be defined as the fraction of its solvency risk dependent on the cumulative solvency of its neighbors: $r_i = \frac{Solvency_i}{\sum_j Solvency_j}, j \in \{j|E^{ij} > 0\}$.

As mentioned earlier, the vulnerability weight measures an institute's ability to control the transmission of stress to its counterparties. Hence, we can characterize the economic value of the impact of one institute on others by the relative solvency condition of the institute with respect to its counterparties.

We define a systemic impact index as a metric to measure systemic risk by taking into account both common idiosyncratic shocks and contagion through counterparty exposures. This index helps rank institutes with a systematic view of their contribution to the transmission of stress through the financial system.

Definition 3. ***Systemic Impact Index***
Similar to Battiston et al. (2012), we define the systemic impact index as a measure of the expected loss generated by the collapse of a financial institution (i) as well as the financial stress that has been received from its counterparties. It can be expressed as

$$x_i = \alpha \sum_j w_{ij} x_j + \beta \sum_j w_{ij} r_i \qquad \alpha + \beta = 1, \tag{7}$$

where α and β are the fractions of stress imposed by a single firm and the entire system, respectively.

Intuitively, the systemic impact index for a financial system represents a specification of the stress induced by each of the nodes in the financial system; this stress is associated with the distribution of capital among the nodes. The total interbank liabilities of a node should never exceed the available capital buffer. Each node must pay off its outstanding interbank obligations equally. Relatively, if a default occurs, all counterparties are paid by the defaulting node equally.

To find an equilibrium and establish a fixed-point characterization of the systemic impact index, we rewrite Equation (7) as a matrix from Sadoghi (2015):

$$x = T^T x, \tag{8}$$

where matrix T is expressed as $T = \alpha W + \beta W r$ and vector x represents the ranks of SIFIs. In the following proposition, we present a successive approximation method for computing x as the rank of SIFIs.

Proposition 1. ***Systemic Impact Index***
In equilibrium, the systemic impact index for each node of a (directed) interbank network can be determined as a solution of the fixed-point iteration

$$x_{k+1} = \frac{k}{k+1} W x_k + \frac{1}{k+1} W r_1,$$

where k is the number of iterations needed to monotonically converge to a fixed point.

The proof is given in the appendix.

To construct a solution, we use a successive approximation to find a fixed point, and state and prove the existence and uniqueness of the equilibrium by means of the contraction mapping theorem.

Definition 4. *Let X be an economy (financial system) defined as a set of firms ($x \in X$) and let $\Phi : X \to X$ be a continuous transformation function that maps X onto itself. A fixed point of Φ is an element $x \in X$ for which $\Phi(x) = x$. We use the notation $\Phi(x) = x$ in place of $x = T^T x$.*

The main problem is to define sufficient conditions on Φ and T to ensure the existence and uniqueness of a fixed point and to find procedures for the calculation of fixed points. To compute the successive approximations and prove the existence of a fixed point, we use an iterative scheme: $x_{n+1} = \Phi(x_n)_{\{n=1,2,\cdots\}}$. Using the Brouwer fixed-point theorem, we prove the existence and uniqueness of the systemic impact index in a banking network.

Proposition 2. ***Existence and Uniqueness of the Systemic Impact Index***
In a financial system with N firms with interbank activities being represented by matrix T and by a continuous transfer function Φ,
(a) there exists an index associated with the ranking of systemically important firms in the network;
(b) transfer function Φ has a unique fixed point in matrix T.

The proof is given in the appendix.

4. Mechanism of Financial Contagion

In this section, we review the dynamics of cascade models in an interbank system and explain the cascade mechanism we use in our simulation study. A prototypical cascade model is based on a number of financial institutions interconnected through directed interbank activities. This procedure creates a series of finite steps until the "cascade equilibrium" emerges as a steady state. Hurd (2015) defines the "cascade equilibrium" as a state of the system with no newly insolvent (defaulted) financial institutions. This equilibrium can be a partial equilibrium depending on the network structure.

In an interbank system, either shocks can spread throughout the entire system or a part of the same in a domino fashion due to interlinked financial exposures or common shocks caused by exogenous factors affecting the whole system. The stress caused by counterparty default may be strengthened by the indirect asset price contagion in an illiquid market. Consequently, the insolvency of an institution causes write-downs in the value of assets held by its counterparties and might lead to a default. A combination of these channels causes either idiosyncratic or aggregate shocks and threatens the stability of the system.

Hurd (2015) categorized cascade mechanisms into three classes and characterized their common features. The first type of cascade, called "default cascade," explains how the default of an individual institution might impact its counterparties, trigger a new default, and lead to a systemic collapse of the entire economy. In the second type of contagion, an illiquid institution might call its loans from its debtors, and thereby impose liquidity shocks on its counterparties and lead to a liquidity cascade in the entire system. The third type of cascade, called "asset fire sales," is observed when the stress caused by counterparty default can be strengthened by an indirect asset price contagion, mainly in an illiquid market. Consequently, the insolvency of an institution causes write-downs in the value of the assets held by its counterparties, and might lead to a default.

As mentioned above, insolvency occurs when a bank's net worth is reduced to zero, that is, when its total losses exceed its capital buffer ($\sum_j Eij_t > K_t^i$). Thus, the solvency condition in a default cascade can be expressed as

$$\sum_j E_t^{ij} > (A^F)_t^i + (1 - h - h_i)(A^C)_t^i + (A^L)_t^i - (L^D)_t^i.$$

The default cascade is started by an individual insolvent institution or a set of insolvent institutions and continues in a domino fashion because of interlinked financial exposures. This procedure can be modeled by updating the balance sheets of affected counterparties at each step. The capital reserves of the nodes of the network are defined as $K_t^j, j \in \{1, \cdots, N\})_t \geq 0$; these are updated as follows:

$$K_{t+1}^j = (K_t^j - \sum_{i \in \Psi(K)} (1 - R_i) E_t^{ji})^+, \tag{9}$$

where R_i is the recovery rate of the defaulting institution i, and Ψ is the set of insolvent institutions given as

$$\Psi(K) = \{j \in V_{\{1,\cdots,N\}} | K_{t-1}(j) = 0 \cup K_{t=0}(j) = 0\}. \tag{10}$$

Ψ includes the group of insolvent institutions at the initial step of the procedure, as a result of the primary cascade. This set will be updated in each step with newly defaulted institutions.

5. Financial stability

After the recent financial crisis, economic stability and related issues have received a lot of attention. It has been argued that to understand the stability conditions of the financial system, a variety of factors underlying the systemic crisis need to be considered. Fluctuations in financial markets, group failures of financial and non-financial institutions, lack of intermediation services, and other issues have systemic consequences. The health status of financial institutions alone does not indicate the stability of a system, and similarly, individual defaults do not lead to financial instability. Therefore, it is essential to clarify the concept of financial stability and its underlying factors.

The scope of the definition of financial stability is broader than the banking system. The stability of a financial system is associated with economic growth, which is facilitated by the financial sector providing financial services on a continuous basis. The potential problems that have a substantial impact on the economy are the central elements of financial instability. There is no standard method to quantify and assess the stability of a financial system. As mentioned earlier, interbank contagion and common shocks caused by correlated illiquid assets are major sources of systemic risk. Nevertheless, it is not well understood which of these sources contribute more to the instability of a system. New insights into systemic crises show that different factors threaten financial stability. Thus far, no comparison of the different systemic risk characteristics in a particular model has been conducted in the economic or banking literature.

A recent study of the stability of the financial system by Acemoglu et al. (2015) introduced a game network and defined financial stability as the inverse of the expected number of defaults in a financial network. The authors define the resilience of a financial system as the inverse of the highest number of potential defaults. Another study by Gofman (2014) defined stability in the banking system as the efficiency of the financial system subsequent to the group failures of institutions. Some studies attempt to measure systemic stability as an aggregation of institutional-level stability metrics. Nevertheless, these metrics are defined in a closed banking system without considering the critical role of the interconnections of the system. To measure financial stability in a broader perspective, we need to identify factors that have substantial consequences for the financial system.

Jackson and Van den Nouweland (2005) used a non-cooperative game model to explain a stable network. In a stable network, none of the players is unilaterally willing to drop their connections and if one player has the benefit of establishing a new connection, other network players suffer by adding the new link. This type of stability can also be investigated using the sequential game of Currarini and Morelli (2000). The existence of a stable network can be proved with a (subgame perfect) equilibrium and depends on how the stability of the system is modeled. This definition merely considers the value of a link for the players at a particular period. However, in a real system, players might continually reorganize their connections to receive more benefits from the system or to reduce their costs.

Beyond the static modeling of a network, we can consider dynamic characteristics of a network whose links or vertices can be added or deleted. The key properties of a dynamic network are the interconnection and complexity that represent how players share resources among their counterparties. Watts (2003) explained the stability of the dynamic network, using an undirected network model to define the stability of the dynamic network. A dynamic network is a stable network if no connection would ever be added or removed over time. The central question is whether different nodes, when they reorganize their connections, have a similar effect on other players' performance.

The recent economic crisis has highlighted that it is essential to understand the network structure of the financial system, and the role of counterparty risk when assessing systemic risk. The triggering of a cascade of counterparties' failures in the financial network depends on the structure of the system and its dynamical properties. To study the structural properties of a dynamic network, we use standard statistical measurements of networks. The connectivity of a network can be measured by the ratio of actual to potential links in a dynamic network. The portion of links that do not alter between two adjacent periods reflects the partial stability of the network. The out-degree and in-degree of nodes capture how nodes are connected, in the interbank network; these properties represent the degree of mutual lending between banks. Other properties like betweenness and closeness, centrality, and diameter measure the connectivity and centrality of the network. In an interbank network, these properties measure the extent to which a bank lies "between" the others in the flow of interbank activities and how fast it can reach all other nodes. Other properties such as clustering and modularity quantify the decomposable characteristics and strength of the division of a network (these are briefly explained in the appendix).

Empirical studies of banking systems show that the relationship between financial stability and the degree of connectivity is not monotonic and depends on the size of interbank liabilities. A more connected banking system is more fragile to contagious defaults, and better-capitalized banks are more resilient to larger systemic risk. However, the relationship between the probability of contagion and level of capital is not linear. The May–Wigner stability theorem May (1972) states that the stability of a large complex system can be inferred from its size and density and from the interactions between its elements and that increasing the complexity of the system leads to a decrease in stability. The statistical properties of an interbank network could help measure its stability and identify circumstances under which macroprudential regulations could mitigate systemic collapse.

There are different academic views on the relationship between the structure of the financial network and systemic risk. Some studies like Allen and Gale (2000) and Freixas et al. (2000) suggested that the incompleteness of a financial system might be a source of its instability, that is, a more complete financial network is a more stable system. A study by Castiglionesi et al. (2010) showed that a system with more dense financial integration is more stable to any individual bank's insolvency. The result of a study of the UK financial system by Haldane et al. (2009) showed that a financial network with a higher degree of connectivity may be a "robust-yet-fragile"? system. In a certain range of connectivity, the interconnection enhances the robustness of the system and absorbs the shocks; however, beyond that range, it serves as a means for the spread of shocks through the system.

On the contrary, other studies showed that more interconnections destabilize the system and contribute to its fragility. Blume et al. (2011) compared the interbank system with an epidemic system and concluded that the likelihood of a systemic failure increases when the number of counterparties grows. In a similar way, Vivier-Lirimont (2006) argued that the magnitude of the spread of negative shocks increases with the density of the interconnected system, and that density can be deemed a mechanism for the propagation of shocks. In summary, financial interconnection has a key role in the stability of the system, but a higher number of connections is not a guarantee of stability.

To study the dynamics of the financial system, we need to build a link between the dynamical properties associated with a financial system and the relationships among network players. We use a model of the banking system and investigate the stochastic stability of interbank activities. Applying the stability theory of dynamical systems, we can assess the stability of a financial system. The stability of a financial system can be understood via the stability of its fixed point in equilibrium. In fact, the stability of a financial system as a dynamic system can be measured by changing the structure of the network over time.

We develop a dynamic model of a banking system to analyze the impact of different macroprudential policies on financial stability and to support decision-making processes. This dynamic model can be used to explore the behavior of complex financial systems, particularly the nonlinear interactions of network structures and system stability. This model represents a system that evolves over time as a result of internal and/or external actions; further, the model's outputs change with perturbations in its inputs. The model allows us to draw conclusions on the robustness of the system against perturbations from unexpected events, environmental changes, and whether changes, enabling us to improve its stability and robustness. It can also be used to identify the weak points of the system that might contribute to large variations in the system's stability.

To measure the dynamical stability behavior of a banking system, we use the information about the system that we used to identify SIFIs. Matrix T in Equation (8) captures the exposure information of a network associated with the conditional status of a financial institution and the status of its counterparties. In our network model, based on the underlying factors of the solvency condition of a bank, the vulnerability weight of a node is defined as the reduction in capital due to the solvency risk from its counterparty: in other words, the health status of a bank depends on the status of its counterparties. The defined solvency condition can measure the level of debt held by a bank. When the debt ratio is high, the firm holds much debt relative to its assets. The lesson from the recent global financial crisis is that the build-up of extreme leverage can facilitate a systemic crisis in the banking system.

We define financial leverage (expressed as leverage multiple) of a bank using capital reserve updating information in a counterparty network (Equation (9)). Index $\Upsilon^i(t)$ measures the financial leverage of company i at time t. This index determines the capability of an individual institution to meet its debt and other obligations at time t with the capital reserve at time $t-1$:

$$\Upsilon^i(t) = \frac{K^i_{t-1}}{\sum_j E^{ij}_t + \sum_{j\in\Psi(i)}(1-R_j)E^{ij}_t)^+\varepsilon^i_t}. \tag{11}$$

The Basel III framework (Committee et al. (2010)) introduced a straightforward non-risk based leverage ratio to determine risk-based capital requirements. This leverage ratio, known as leverage multiple, expressed as

$Capital/Exposure$, where capital is the Tier 1 capital of the risk-based capital and exposure can be determined via summation of the on-balance sheet exposures (e.g., collateral assets), off-balance sheet items, derivative exposures (e.g., credit risk), and securities financing transaction exposures. The non-risk-weighted leverage ratio requirement is a supplement of the risk-weighted capital requirements and has implications for banks' stability. It must be at least 3% according to Basel III (Committee et al. (2010)). The non-risk-weighted leverage ratio provides a more robust capital buffer in times of crisis and enhances system stability. Blum (2008) analyzed risk-weighted capital requirements and showed that an additional risk-independent leverage ratio control might be essential to recognize dishonest banks and improve banking supervision.

The leverage of a financial system is defined as an aggregation of banks' leverage at the individual level. The aggregate asset values, interbank exposure, network structure, and leverage might change over time. Shin (2009) introduced a leverage ratio for the whole financial system by aggregating leverage levels of individual banks. In the following lemma, we define the financial leverage of a system by means of network properties.

Lemma 5.1. ***Financial System Leverage Index***
Financial system leverage as a function of the aggregate level of the asset values, interbank exposure, and the network structure can be expressed as

$$\bar{\Upsilon}(t) = \varpi_t \frac{(\bar{A^L})_{t-1} + (1-h)(\bar{A^C})_{t-1} + \bar{d}_{t-1}E^{max}_{t-1} - (\bar{L^D})_{t-1}}{(\bar{d}_t + R(\#default)_t)E^{max}_t + \bar{\varepsilon}_t}, \tag{12}$$

where

- $(\bar{A^L})_t$ *is the average value of liquid assets in the system at time* t*;*
- $(\bar{A^C})_t$ *is the average value of collateral assets in the system at time* t*;*
- E^{max}_t *is the maximum exposure in the system at time* t*;*
- $\bar{d}_t$ *is the average degree of nodes at time* t*;*
- $(\bar{L^D})_t$ *is the average deposit at time* t*;*
- $(\#default)_t$ *is the expected number of failed banks at time* t*;*

- ϖ_t *is the relative number of active banks changing over time;*
- $\bar{\varepsilon}_t$ *is the average deposit shock at time* t.

The proof is given in the appendix.

Bair et al. (2015) argued that the leverage ratio might be the most useful financial stability tool alongside risk-based capital requirement to enhance the resiliency of a financial system during downturns. The stability of a financial system can be affected directly by high leverage as this would magnify the distress caused by a bank's failure and transmit the same to many other institutions. Therefore, measuring and restricting leverage at the system level is necessary for reducing systemic risk.

The financial system leverage index $\bar{\Upsilon}_t$, which is defined in Equation (12), shows that leverage has a nonlinear relation with the average degree of connectivity of the network, $\bar{d}_t$. It also has an inverse relation with the expected number of defaulted banks. The ratio of the number of active banks between two adjacent periods ($\bar{\Upsilon}_t$) reflects the partial stability of the network over time.

This result is consistent with earlier findings on the stability of large systems, such as May (1972) and Gofman (2014), who show a non-monotonic relation between the highest degree of the nodes in the financial network and its resilience.

In equilibrium, a firm trusts its relations with its counterparties and keeps the required capital to absorb the stress from the system. When the system is moved away from equilibrium, a restoring force pushes it back to a stable equilibrium point. We investigate the efficiency of equilibrium using the characterization of financial contagion through endogenous interbank counterparty relations. The main endogeneity in this model includes the structure of the network and the terms of mutual interbank agreements. We define equilibrium by using the non-cooperative game model, which was introduced by Jackson and Van den Nouweland (2005).

Definition 5. ***Cascade Equilibrium***
The system is in equilibrium when it satisfies the following two conditions:

1. *No player in the system is unilaterally willing to drop its connections with its counterparties. This means that the expected profit each counterparty receives from its relation is not smaller than the expected profit (minus risk exposure) by ending its relations.*
2. *Two players who are not linked through the interbank network have no incentive to establish a relation. This means that if one player has a benefit from forming a new link, the other player in the proposed link will suffer a cost.*

Let $\bar{\Upsilon}(t)$ be a non-linear, time-varying dynamic system of financial system leverage, given as

$$\dot{\bar{\Upsilon}}(t) = f(\bar{\Upsilon}(t), u(t), t), \tag{13}$$

where $u(t)$ is the input function. Let $(\bar{\Upsilon}_{t_0})$ be the financial system leverage at the initial state.

Let new state $\eta(t) = \bar{\Upsilon}(t) - \bar{\Upsilon}_e$ be the distance between the current state of the system and an equilibrium state. Using a Taylor series expansion and computing derivative $\eta(t)$ with respect to the expectation of interbank exposure, we can show that

$$\dot{\eta}(t) = \frac{\bar{d}_{t-1}}{\bar{d}_t + R(\#default)_t)}\eta(t). \tag{14}$$

Equation (14) shows that to assess the resilience of the financial system, we need to take into account the altering configuration and the dynamics of the system. Based on this result, we can define the stability condition of a system as follows.

Definition 6. ***Stable System***
A system is stable with respect to perturbations of the initial state (cascade) and input function (contagion) if

$$||\delta\eta(t)|| < \varepsilon, \qquad \forall t_0, \varepsilon, \quad t \geq t_0, \tag{15}$$

where

$$\delta\eta(t) = (\eta(t_0) + \delta\eta|_{t_0}, u(t) + \delta u(t), t) - \eta(t_0) \tag{16}$$

is the system variation, when moving away from an equilibrium state due to the perturbations of the initial state (cascade) and the input system (contagion).

The system might converge to the equilibrium state in the long run if

$$\lim_{t \to \infty} ||\delta\eta(t)|| = 0. \tag{17}$$

The ultimate goal of regulation is to build a banking system that can absorb stress and resist perturbations caused by unpredicted large-scale events. The desired system should have the ability to recover to a stable state and provide acceptable quality services as rapidly and economically as possible. This system should restore its functionality and maintain its core purpose for a long period. We assess the resilience of the banking system with respect to the dynamics and structure of an interbank market over time.

Definition 7. ***System Resilience***
The banking system can change dynamically over time, such that the formation of this system can evolve based on outside events and/or the decisions made by agents. The resilience of this system can be measured by the change in its leverage over time:

$$\zeta(t) = \int_{t_0}^{t} [\bar{\Upsilon}(t_0) - \bar{\Upsilon}(s)]ds. \tag{18}$$

We assess the stability of a banking system affected by the different mechanisms of stresses by measuring the financial system leverage ratio. Knowing which macroprudential policies can minimize losses and damages, and maximize the ability and capacity of the system to adapt and adjust to such events, is beneficial. We measure the resilience of the system in terms of its ability to find adaptation strategies corresponding to crises and to return the system to a functioning condition. It is essential to understand the post-shock system performance and design a recovery mechanism for a system to provide acceptable high-quality service over time.

6. Macroprudential policy

In this section, we consider common issues associated with regulating financial institutions, mainly banks. We study different macroprudential policies to reduce transmission of shocks in the system. To improve financial stability, a systemic view on the resilience of the financial system needs to consider the financial system as a full complex system. Macroprudential supervision is directly interlinked with conventional supervision. Later, we analyze the regulatory rules imposed on the system and discuss how the new regulatory mechanisms respond to systemic events.

Banking regulation is a form of government intervention into the financial activities of banks by imposing rules or legal restrictions. These regulations include a set of standard settings for capital, leverage, and liquidity requirements. These regulations carry costs and benefits for the entire economy. The benefits are derived from the reduction in expected crisis costs and the increase in the resilience of the economy. Costs are associated with the impacts on lending spreads and the costs of inappropriately regulated leverage and capital that might contribute to the initiation of financial crisis (see, e.g., Elliott et al. (2012)).

Microprudential policies consider different mechanisms to insure individual institutions against default. Insurance companies or the private sector in capital markets provide this guarantee and diversify the risk of individual institutions. The traditional function of central banks is to act as a Lender Of Last Resort (LOLR). Central banks can stabilize the system by lending to illiquid but solvent institutions or to insolvent institutions against an uncertain collateral.

In the aftermath of the recent crisis, practitioners and academics admit that the microprudential policies failed to appropriately reduce the consequences of systemic crises. Indeed, these policy interventions in the financial system are a set of individual-level decisions that did not take the risk of counterparties into account. The microprudential policies regulate banks to diversify their stress with the objective of enhancing individual-level stability. These policies amplify the likelihood of a systemic collapse, since they increase asset return correlations as one of the primary sources of systemic risk. On the contrary, a macroprudential approach has a systemic viewpoint and seeks the stability of the entire financial system.

In what follows, we review macroprudential tools to mitigate systemic risk. One way to mitigate systemic risk is to require an ex-ante capital buffer. A counter-cyclical buffer is a well-understood tool that is specified in the microprudential regulation. It is required that banks hold a sufficient amount of equity to secure creditor loss. This rule increases the capital ratio by shrinking assets and indicates a shift away from system equilibrium. From this viewpoint, the higher **minimum capital requirements** imposed in Basel III raise asset correlations and lead to higher systemic risk. Therefore, minimum capital requirements should be continually adjusted for non-SIFIs in non-crisis times.

Imposing minimum capital requirements can regulate the liability side; alternatively, the regulator can adjust the asset side by imposing higher minimum liquidity requirements. By increasing banks' stress-free liquidity of banks, policy makers provide self-insurance with regard to paying back debt to creditors in the short term. Nevertheless, this regulation regime might amplify the reliance on liquidity with liquid assets that are eligible for central bank refinancing.

The lesson from the recent global financial crisis is that generally both leverage and liquidity have significant contributions to systemic collapse, and thus regulating both is necessary Turner et al. (2009). The **leverage ratio**, commonly used as a measure of regulatory leverage, is a complementary instrument to the minimum capital requirement tool. D'Hulster (2009) elaborated on the characteristics of the leverage ratio and demonstrated the restrictions on its implementation. This regulatory tool at the micro level can be used as a leverage limit for supervised entities. Moreover, as a macroprudential tool, it can be a gauge for monitoring the susceptibility of a system and as a trigger for capital requirements, as defined under Pillar 2 of Basel II.

Individual- and aggregate-stress-based haircuts help preserve the capability of banks to pay back debt. Fecht et al. (2015) documented the overall reduction in borrowing collateral from the Eurosystem by German banks after Lehman's default. Their empirical results indicated an improvement in the average quality of their collateral: they were holding more collateral relative to the liquidity of the Eurosystem, given that solvent banks were obligated to provide extra collateral.
A general difficulty when employing macroprudential tools is setting regimes tightly in non-crisis periods, while reducing the vulnerability of the system in crises.

7. Model Simulation

This section outlines a framework for deriving policy implications from our model. We use a numerical simulation to investigate the role of macroprudential regulation to stabilize the financial system. We simulate our model to detect SIFIs and contributions of individual institutions' stress to the entire system. Our simulation is based on a fully heterogeneous model of an interbank system. We assess the stability of the financial system by means of network properties and how the network properties change dynamically through time. Our model characterizes a stylized bank's balance sheet and its interconnection to its counterparties.

Our simulation of the model starts from the current state of the system, that is, it is not necessary that the system is in equilibrium. At time zero, all institutions are in the normal state, and there is no defaulting bank in the system. The balance sheet will be updated dynamically following a mark-to-market procedure. We suppose banks just have access to interbank resources and no external sources such as central banks. The simulation of exposures is independent of network properties that would raise questions about the validity of the system. Finally, we assume that at the time of default, solvent nodes impose stress evenly on their counterparties.

We investigate different types of cascades, including defaults by one or several random institutions, defaults by SIFIs, and common shocks to the entire system. Later, we introduce different macroprudential policies to mitigate systemic collapse. We also analyze the resilience of the financial system to contagion that begins with a collapse of the most interconnected institution (SIFI) and causes a cascade of counterparty defaults.

The network of interbank counterparties can be modeled as a random graph to describe how financial contagion is propagated. The random graph families that have been used in the literature are inhomogeneous random graphs, for example, Erds–Renyi random graphs (Erdős and Rényi (1959)) and preferential attachment models. Albert and Barabási (2002) suggested a preferential attachment model of an undirected graph with a power-law distribution. Bollobás et al. (2003) proposed a directed scale-free graph with preferential attachment associated with the degree distribution of the graph. In our simulation, we model interbank networks with a directed scale-free graph proposed by Bollobás et al. (2003), which is similar to that introduced in Amini et al. (2012b). In this model, when a new firm enters the market and starts to establish relations with current firms, it is more likely to choose highly connected firms as counterparties.

We simulate an interbank exposure network using the parameters derived in an empirical study by Cont et al. (2010). They studied the Brazilian financial system and observed that this financial network is a complex heterogeneous network with heavy-tailed in-degrees and out-degrees distributions. The mutual exposure of counterparties in this financial system is distributed as a power-law tail with exponents between two and three during crises. They observed a heterogeneous and complex system with correlated degrees associated with the size of liabilities and exposures. Another study by Boss et al. (2004) of the Austrian financial system showed similar network properties, for example, degrees following power-law distributions and mutual exposures following Pareto distributions. The consistency in the empirical findings of the studies on two different financial systems led us to simulate our financial random network with the aforementioned random-graph properties.

The recent financial crisis showed that banks need to increase the quantity and quality of capital to improve the stability of the system. Capital comprises a cushion to prevent likely losses. Basel III restricts common equity Tier 1 to above 4.5% of risk-weighted assets and minimum Tier 1 capital to above 6%. The total capital must remain at a level of at least 8% of risk-weighted assets. Furthermore, banks are required to hold high-quality liquid assets, especially during times of stress.

In our simulation, we follow the balance-sheet configuration of Gai et al. (2011). In this setup, liquid assets and collateral assets are 0.1% and 10% of the balance sheet, respectively, and total capital is 6 percentage points of risk-weighted assets. For different simulation exercises, we adjust the proportion of both capital and liquid assets.

We simulate a directed scale-free graph explained by Bollobás et al. (2003) using parameters derived from a real financial network. Figure 1 shows a simulated direct random network. We identify SIFIs in the network using the systemic impact index. The nodes of this weighted network represent systemically critical nodes that can threaten the stability of the system.

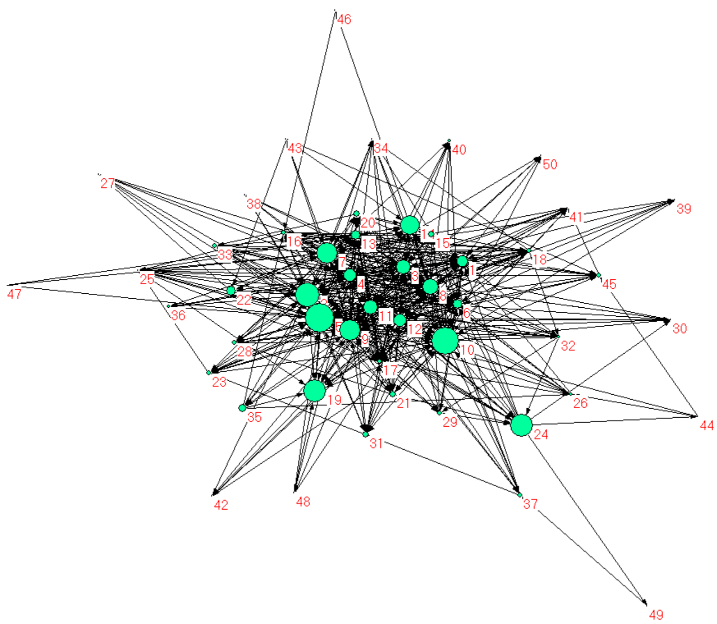


Figure 1: Visualization of a simulated interbank network.

7.1. Experiments

We set up a framework for deriving policy implications and examining how different policies can help avoid a systemic collapse. We formulate policies and design corresponding experiments with the initial evaluation of the possibility of implementation and assess the impact and effectiveness of policies in achieving stated goals. The fundamental parameters and variables of the model are evaluated as to whether they are amenable to change through policy intervention, and if so, whether it would be possible and reasonable in practice. Prior to making policy recommendations, the ethicality of altering the variables is considered.

7.1.1. Experiment 1: Basic-exposure interbank network

In the first experiment, we simulate a stylized balance sheet with 80% non-banking assets and 20% interbank assets. The non-banking asset side of the balance sheet consists of 2% liquid assets and 10% collateral assets, with the remaining being fixed assets. The total capital buffer must remain at a level of 4% of risk-weighted assets. The initial aggregate haircut of the system is 0.1%, and in this experiment, there are no individual haircuts for any bank. We consider both zero-recovery and positive-recovery rates for counterparties of a default node. We simulate an interbank network as a directed scale-free graph and mutual exposures with parameters mentioned in the empirical study by Cont et al. (2010). In this experiment, we shock a single random node by considerably stressing its exposure. This node starts to hoard liquidity and impose stress to its counterparties.

7.1.2. Experiment 2: Shocking a single SIFI

In this experiment, we use the same structure of the balance sheet and the same interbank network. To study the difference between the propagation of shocks started by a SIFI or a random node, we identify a SIFI in the first stage. We stress the identified SIFI with a large individual haircut shock.

7.1.3. Experiment 3: Common shock model

As mentioned before, stress can either spread throughout the entire system or a part of the same in a domino fashion due to interlinked financial exposures or exogenous factors have a strong influence on the entire system. In this experiment, we keep the basic configuration of the balance sheet and interbank network, and the rate of aggregate haircut as in experiment 1. Here, we shock the entire system by shocking all nodes that receive adverse individual haircut stress to create the liquidity-hoarding phenomenon.

7.1.4. Experiment 4: Different levels of capital buffers for all banks

In experiment 4, we examine the stability of the system with a different capital buffer for all banks. We use the same balance sheet and network structure, and change the level of capital required of banks.

7.1.5. Experiment 5: Different capital buffers for SIFIs and non-SIFIs

New regulations of financial institutions require SIFIs to have contingency plans and higher capital or liquidity levels as safeguards against possible systemic failures. These regulations try to prevent SIFIs from becoming "too big to fail" or "too connected to fail." One way to mitigate systemic risk is to require different capital buffers for SIFIs and non-SIFIs.

7.1.6. Experiment 6: Adding individual and aggregate haircut shocks

In this experiment, we increase the initial haircut by introducing individual haircuts for SIFIs and assess the behavior of the system with respect to this parameter. A haircut is a portion of an asset's value that is used as collateral. Haircut size is typically associated with the liquidity risk of the asset to be used as collateral. Higher haircuts lead to safer positions for lenders and protect them against losses incurred when selling collateral because of counterparty defaults.

7.1.7. Experiment 7: Having leverage ratio requirement

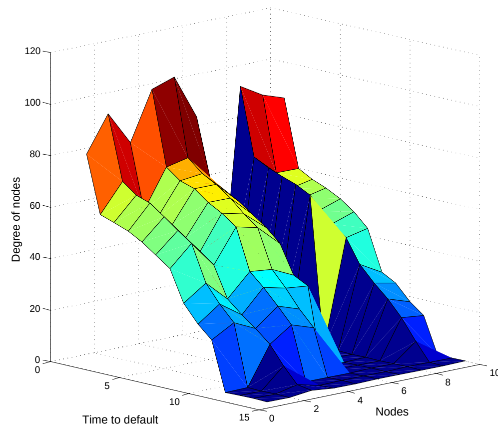
As mentioned earlier, leverage ratio requirements can be applied as a microprudential policy as well as a macroprudential tool. The simplest way to calculate the leverage ratio is as the minimum proportion of Tier 1 capital to entire regular adjusted assets. It is recommended that banks having a leverage ratio of at least 4% for all banks and 3% for banks if their risk profiles warrant them. In this experiment, we apply macroprudential policies and use the same structure of the balance sheet and the same interbank network as used in the first experiment. The propagation of shocks was started by a SIFI.

7.1.8. Experiment 8: Having different leverage ratio requirements for SIFIs and non-SIFIs

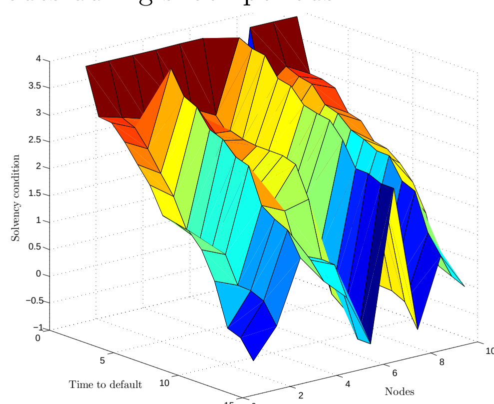
This experiment is similar to experiment 7; however, we maintain different minimum leverage ratio for SIFIs and non-SIFIs. A higher leverage ratio may be required for a financial institution if it requires closer supervision due to managerial or operational weaknesses. We maintain a minimum leverage ratio of 5% for SIFIs, to be considered as well-capitalized institutions and also reducing the propagation of shocks.

7.2. Discussion and Policy Implications

An insolvent node in the banking system can pose systemic risks to the whole system. Figure 2 shows the solvency condition for individual institutions, which indicates the situation when debtors start collecting debts. This plot also shows the decline in the number of connections during the shock period. Therefore, insolvent institutes have fewer counterparties to diversify their stress.



(a) Distribution of the degree of selected nodes during shock periods.



(b) Decreasing the solvency condition of banks in time.

Figure 2: Experiment 1: Changing the degree of individual nodes in the network and their solvency conditions until the cascade of liquidity hoarding ceases.

The results from empirical studies show the positive correlation between the size of average interbank assets and average interbank liabilities of a bank. In an interbank network topology, the in-degree and out-degree of a node represent the average sizes of interbank assets and liabilities, respectively. Figure 3 shows the distribution of degrees during the liquidity-hoarding period.

Looking at the graphs displaying the average in-degrees and out-degrees of nodes confirms the significant role of SIFIs in the propagation of stress in the system.

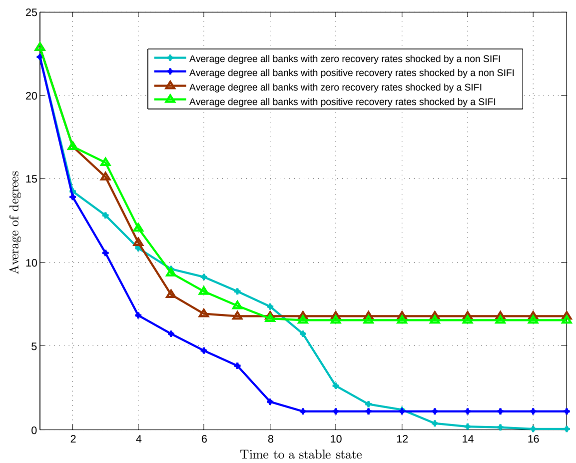
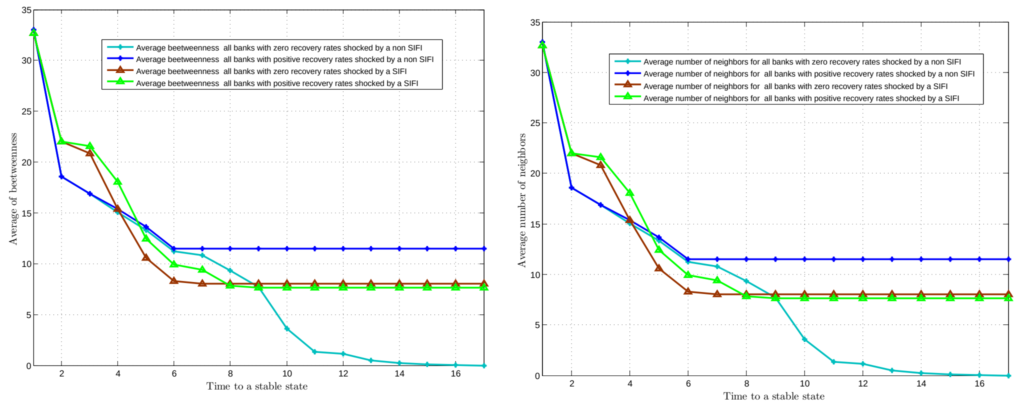


Figure 3: Average degree of the network until the liquidity-hoarding cascade ceases.

Another important centrality measurement of a network is the minimum number of pairs of individual nodes that have to be passed to reach another node. This indicator shows the complexity level of a network regarding the possibility of decomposition into smaller communities. Studies of different types of networks in social sciences show that a decomposable system would be stable in time. Figure 4 shows the change in average betweenness and the average number of neighbors at each node, that is, the system becomes less decomposable than before. As a result, the system moves away from the equilibrium to an unstable state.

Figure 5 shows the percentage of defaults caused by shocks to SIFI and non-SIFI nodes. We remark that a higher level of capital buffer can reduce the vulnerability to a systemic collapse. Banks decide on the optimal capital level by examining how much capital must be provided to prevent solvency risk. After the 2007–2008 economic crisis, academics and practitioners ac-



(a) Average of betweenness. (b) Average number of neighbors.

Figure 4: Experiments 1 & 2: Average betweenness and number of neighbors in the network the liquidity-hoarding cascade ceases.

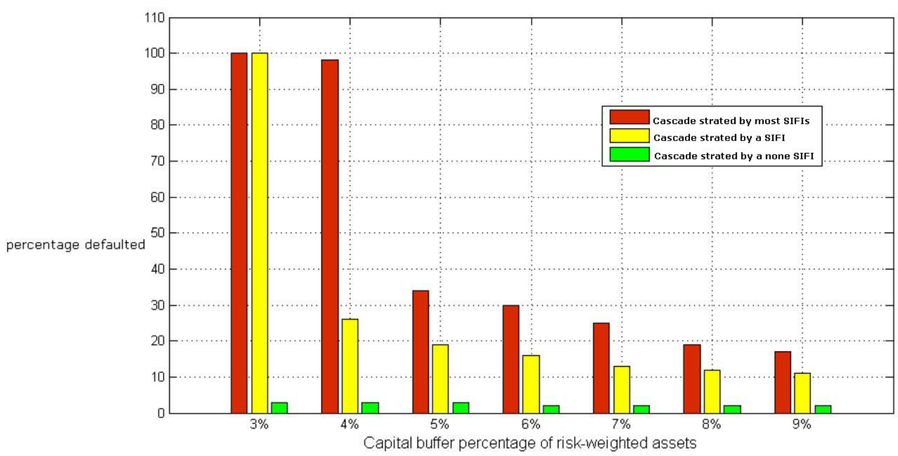


Figure 5: Number of defaults associated with different capital levels for banks.

knowledge that this level must also account for counterparty risk.

We analyze the different required capital levels to absorb stress. Plotting the distribution degree and the number of neighboring nodes (Figure 6) gives that an increase in the level of capital can improve system stability. Banks decide on the optimal capital level by examining how much capital must be provided to prevent solvency risk. Basel III changes the form of banks' capital and increases the minimum level to 4.5% of assets while keeping the additional 2.5% buffers to secure the system during a financial crisis.

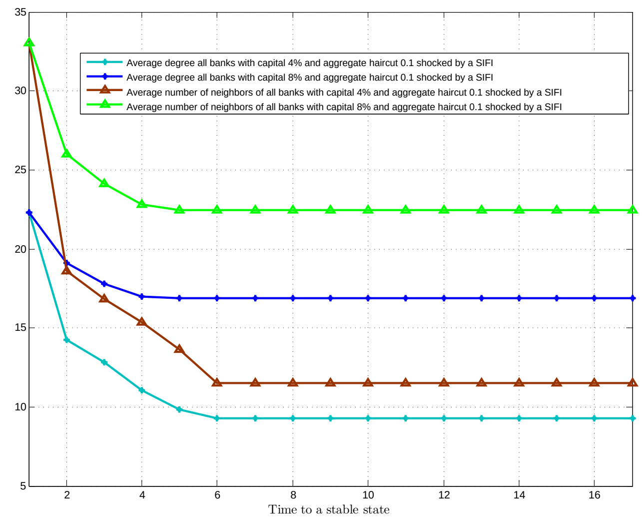


Figure 6: Average degree and number of neighbors when the entire system is exposed to a shock, under different capital levels.

In experiment 5, we simulate the system under different levels of capital buffers for SIFIs and non-SIFI. Plotting the system financial leverage (Stability) of the network (figure 7) gives that the system is even stable with lower capital buffers, while having a higher level for SIFIs. In equilibrium, a firm trusts its relations with its counterparties and keeps the required capital to absorb stress from the system. New rules change bank and system equilibrium and increase the capital ratio by shrinking assets; on the other hand, these policies reduce the efficiency of banking functions. For that reason, minimum capital requirements should be continually adjusted for non-SIFIs during non-crisis periods.

By plotting the centrality measure of the network (figure 8), we can investigate how different haircut rates for SIFIs and non-SIFIs might restrict the exposure to counterparties and improve the stability of the system. Having

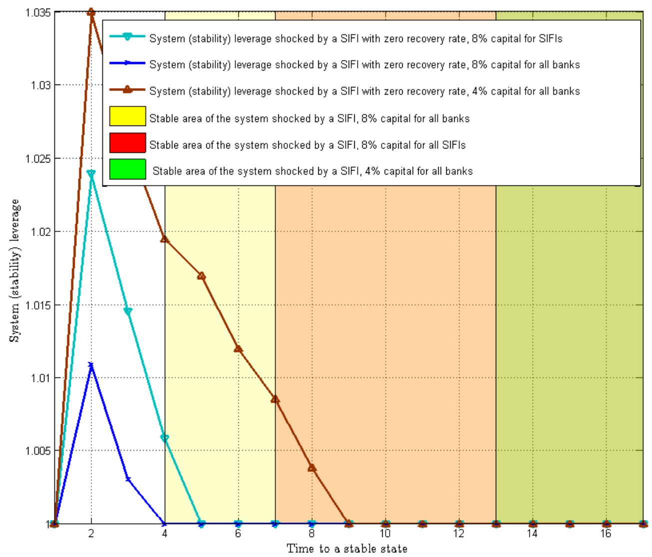


Figure 7: System financial leverage (stability) under different capital buffers for SIFIs and non-SIFIs.

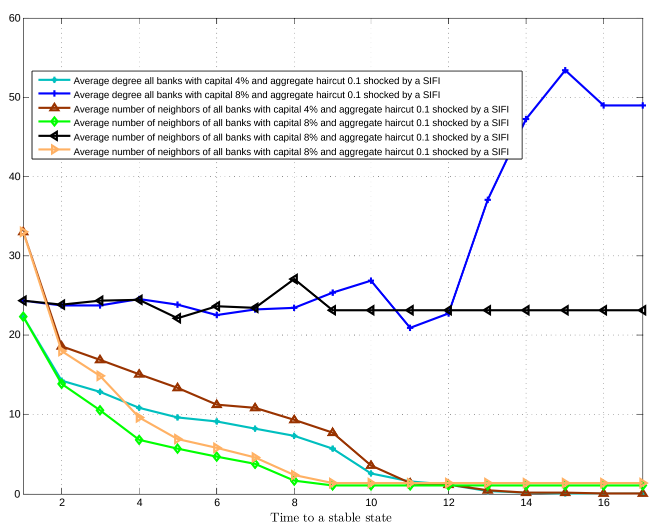


Figure 8: Average degree and betweenness after introducing individual and aggregate haircut rates.

a tight regime during non-crisis periods for all institutions might reduce the vulnerability of the system to systemic liquidity risk in times of crisis. However, different macroprudential policies for SIFIs and non-SIFIs can mitigate the vulnerability to systemic collapse.

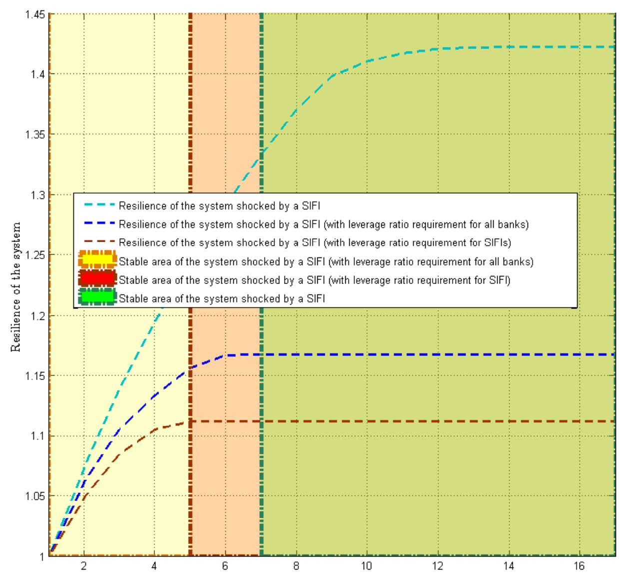


Figure 9: Resilience of a system after a shock to a SIFI, with and without the leverage ratio requirements for all institutions or SIFIs.

Plots of system financial leverage (Figure: 9) and resilience of a system after shocks (Figure: 10) give that minimum leverage requirements can mitigate the propagation of shocks and enhance system stability. This regulation is rather easy to apply and monitor; however, it has limitations due to the lack of risk-based capital requirements, thus requiring transparency for asset risk assessment. Under this regulation, institutions need to hold highly liquid, high-quality securities. Applying different regulations for SIFIs and non-SIFIs can be recommended, but even then, this regulation—as a single tool—cannot prevent systemic crises with certainty.

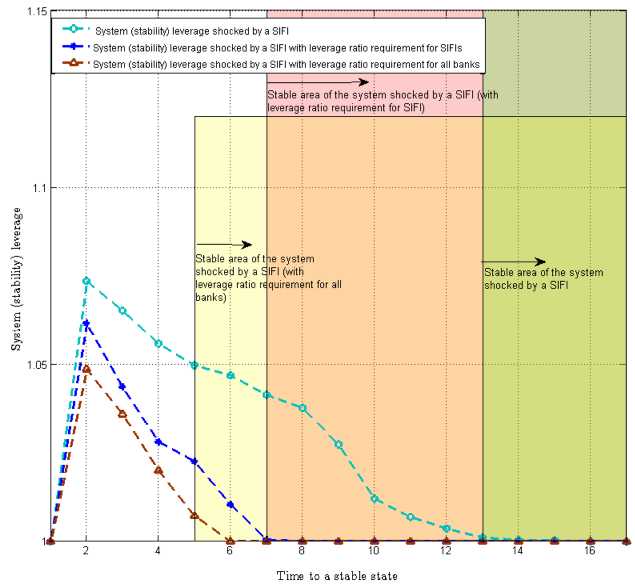


Figure 10: System financial leverage (stability) after a shock to a SIFI, with and without leverage ratio requirements for all institutions or SIFIs.

8. Conclusions and Remarks

In this article, we study the stability of an interbank network as a complex system. We review sources of systemic collapse that were revealed in the recent economic crisis. By means of fixed-point theory, we identify SIFIs by considering the explicit characterization of the equilibrium behavior of the banking system. We discuss how to measure stability in an interbank network by means of the leverage ratio of the system.

The main objective of this article is to provide policy makers with information to allow them to modify or formulate regulatory policies. We try to provide some evidence and raise the question as to how best can the findings of theoretical research be fed into the regulation policy procedure. In an attempt to deal with the issue, we construct a simple interbank framework for deriving policy implications. Our aim is to raise some of the questions that need to be answered and highlight some issues that need to be considered, rather than to give straightforward answers. As a consequence of the absence of consistent information on banking policy interventions and unknown sources of systemic collapse and the complexity of the system's structure, this form of regulation analysis, in practice, is rare. Therefore, it needs to be done in an experimental form with some assumptions and limitations. It is clear that there is a need for innovative policy responses that are more directly associated with the nature of the systemic-risk problems to be solved

In our simulation, we focus on different mechanisms of contagion: common shocks and domino-fashion sequential defaults and combinations thereof started by random institutes or SIFIs. Different macroprudential policy scenarios are implemented to reduce the transmission of shocks to the system. Our numerical results show that it is essential to choose appropriate aggregate or specific macroprudential regimes for SIFIs and non-SIFIs.

The results indicate that the percentage of failures in a financial system depends on the network's architecture and its stability, and also how shocks start and propagate through the system. In other types of networks such as social networks or epidemiology systems, interconnections can speed up the propagation of a phenomenon. In the financial system, interconnections play a fundamental role in amplifying contagion but also in reducing risk via diversification.

On a broader level, we have seen that there are two main findings in the application of our framework. First, the results highlight that none of the existing or proposed regulatory policies—when applied as a single strategy—can stop or mitigate systemic collapses. This means that it is necessary to apply a combination of policies when considering the role of an institution in the system. Second, in reality, policymakers should look at several indicators to identify SIFIs and measure the stability of a financial system to create an early-warning system for systemic crisis. We leave the study of the optimal policy, in terms of cost and benefit, in this type of model for future research.

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Appendix

Proof of Proposition 1

To compute the optimum value of vector x in the equation ($x = Tx$), we define the following least squares error-minimization problem:

$$||\bar{x} - T\bar{x}||_2 = \frac{||T_k x_1 - x_1||_2}{k} \leq \frac{Constant}{k}, \tag{19}$$

where $T = \alpha W + \beta W r$.
Vector x can be expressed as an averaging series of its components:

$$x_k = \frac{x_1 + T_1 x_1 + \cdots + T_{k-1} x_1}{k}, \tag{20}$$

and by choosing the initial value of the iteration procedures as $x_1 = W r_1$, we get the following recurrent form:

$$x_{k+1} = \frac{k}{k+1} W x_k + \frac{1}{k+1} W r_1. \tag{21}$$

Using equation $T = \alpha W + \beta W r$, T_k can be expressed as

$$x_k = \frac{k}{k+1} W + \frac{1}{k+1} W r. \tag{22}$$

Proof of Proposition 2

Part (a): $\Phi : X \rightarrow X$ is defined as a continuous, transformation function that maps X onto itself. Based on Brouwer's fixed-point theorem, Φ is a continuous mapping function. Therefore, there exists a fixed point $x_i \in X$. Vector x is an economy that is a set of firms corresponding to an interbank equilibrium of a financial network. Note that any interbank equilibrium in this model is a fixed point of Φ.

Part (b): If there exist two fixed points x_1 and x_2,

$$d(x_1, x_2) = d(\Phi(x_1), \Phi(x_2)) < \theta d(x_1, x_2), \tag{23}$$

where $d(x_1, x_2) = ||x_1 - x_2||_2$ with condition $0 \leq \theta < 1$. We should prove that $d(x_1, x_2) \rightarrow 0$.
According to the Perron–Frobenius theorem, proved by Perron (1907) and Frobenius et al. (1912), for matrix T (a real square matrix with non-negative

entries), there exists a dominant eigenvector $\bar{x}$ with strictly positive components, which can be expressed as

$$\hat{x} = \underset{x \in \sum}{\operatorname{argmin}} \{||\bar{x} - T\bar{x}||_2 + \epsilon\}.$$

Similarly, we have $Tx = x - \epsilon$, and $\hat{x}$ coincides with $\bar{x}$ if ϵ is small enough. Then, we can obtain a linear convergence in the fixed-point iterations.

Given the mapping function Φ on $X \to X$,

$$\begin{aligned} d(x_1, x_2) &= d(\Phi(x_1), \Phi(x_2)) \\ &= d(Tx_1, Tx_2) \\ &= d(x_1 - \epsilon_1, x_2 - \epsilon_2) \\ &< d(x_1, x_2) - d(\epsilon_1, \epsilon_2) \\ &\leq \theta d(x_1, x_2), \end{aligned}$$

we can write the above equation in a general form

$$\begin{aligned} d(x_n, x_{n+1}) &= d(\Phi(x_n), \Phi(x_{n+1})) \\ &\leq \theta d(x_n, x_{n+1}). \end{aligned}$$

The recurrence form gives us

$$\begin{aligned} d(x_n, x_{n+1}) &\leq \theta^n d(x_0, x_1) \\ (n = 1, 2, ...), & \end{aligned}$$

which implies $d(x_p, x_q) \to 0$ for any arbitrary p, q in $n = 1, 2, \cdots$. Hence, the interbank equilibria of the model are basically unique.

Proof of Lemma 1:

We define the capital reserve of a node (bank) in the interbank network at time t as $(K_t^j), j \in \{1, \cdots, N\})_t \geq 0$; this is updated as

$$K_t^i = (K_{t-1}^i - \sum_{j \in \Psi(K)} (1 - R_j) E_{t-1}^{ij})^+.$$

The solvency condition in a non-default bank at time t, when it receives a deposit shock, can be expressed as

$$K_t^i > \sum_j E_t^{ij} + \varepsilon_t^i.$$

We define the financial leverage (leverage multiple) of a bank using capital reserve and by updating the information from a counterparty network (Equation (9)). Index $\Upsilon^i(t)$ measures the financial leverage of company i at time t:

$$\begin{aligned}\Upsilon^i(t) &= \frac{K^i_{t-1}}{\sum_j E^{ij}_t + \sum_{j\in\Psi(i)}(1-R_j)E^{ij}_t)^+\varepsilon^i_t} \\ &= \frac{(A^F)^i_{t-1} + (1-h_i)(A^C)^i_{t-1} + (A^L)^i_{t-1}\sum_j E^{ij}_t - (L^D)^i_t}{\sum_j E^{ij}_t + \sum_{j\in\Psi(i)}(1-R_j)E^{ij}_t)^+\varepsilon^i_t}.\end{aligned}$$

Now, we define the leverage of the financial system as an aggregation of individual banks' leverage in a financial network:

$$\bar{\Upsilon}(t) = \frac{1}{N_t}\sum_i \Upsilon^i(t).$$

We approximate the average of interbank exposure E^{ij}_t at time t as $\bar{d}_t E^{max}_t$, such that $\bar{d}$ is the average connectivity of the network and E^{max}_t is the expectation of the extreme-value interbank exposure at that time since it has a power-law distribution.

Thus, financial system leverage, when using financial network properties, can be expressed as

$$\bar{\Upsilon}(t) = \varpi_t \frac{(\bar{A^L})_{t-1} + (1-h)(\bar{A^C})_{t-1} + \bar{d}_{t-1}E^{max}_{t-1} - (\bar{L^D})_{t-1}}{(\bar{d}_t + R(\#default)_t)E^{max}_t + \bar{\varepsilon}_t *},$$

where $(\bar{A^L})_t$, $(\bar{A^C})_t$, $(\bar{L^D})_t$, and $\bar{d}_t$ denote the average value of liquid assets, average value of collateral assets, average deposits, and the average degree of the system, respectively. E^{max}_t is the maximum exposure and $\bar{\varepsilon}_t$ is the average deposit shock to the system. $(\#default)_t$ is the expected number of failed banks at time t and ϖ_t is the relative number of active banks in the period from time $t-1$ to time t.

A: Measurement of Network Centrality

Definition 8. ***In-Degree***

In a financial network setting, the in-degree of one node represents the number of its creditors. The in-degree of a node i is given as

$$degree^{in}_i ? = \#j ? W | w_{ji} > 0. \tag{24}$$

Definition 9. ***Out-Degree***
In this network, the out-degree of a node represents its number of debtors, and is given as

$$degree_i^{out} := \#j?W|w_{ij} > 0. \tag{25}$$

The other important characteristics of a network are how close a node is to other nodes and how easy it is to reach other nodes.

Definition 10. ***Closeness Centrality*** *Closeness centrality (CC) measures how far a node is from other nodes when measured in terms of the relative distances between nodes. It can be expressed as*

$$CC_i = \frac{n-1}{\sum_j A_{i,j}}. \tag{26}$$

Definition 11. ***Betweenness Centrality*** *Let P_{ij} be the total number of paths between nodes i and j. Betweenness centrality (BC) measures the intermediary characteristics of a node and can be defined as*

$$BC_k = \sum_{i,j \neq k} = \frac{\frac{P_{ij}^k}{P_{ij}}}{\binom{n-1}{2}}, \tag{27}$$

where P_{ij}^k is the number of paths passing through node k.

B: Basic BA Algorithm

The basic Barabasi–Albert (BA) algorithm for generating random scale-free networks proceeds as follows (cf. Albert and Barabási (2002)):

1. Start with an initial set of small fully connected nodes;
2. Add new vertices one by one, each one with exactly m edges;
3. Each new edge connects to an existing vertex in proportion to the number of edges that the vertex already has a preferential attachment with.